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Adaptive Highlights Stencils for Modeling of Multi-Axial BRDF Anisotropy

An Alternative to Analytical Anisotropic BRDF Modelling

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Abstract Directionally dependent anisotropic material appearance phenomenon is widely represented using bidirectional reflectance distribution function (BRDF). This function needs in practice either reconstruction of unknown values interpolating between sparse measured samples, or requires data fidelity preserving compression forming a compact representation from dense measurements. Both properties can be, to a certain extent, preserved by means of analytical BRDF models. Unfortunately, the number of anisotropic BRDF models is limited, and moreover, most of them require either a demanding iterative optimization procedure dependent on proper initialization, or the user setting of parameters. Most of these approaches are challenged by the fitting of complex anisotropic BRDFs. In contrast, we approximate BRDF anisotropic behavior by means of highlight stencils and derive a novel BRDF model that adapts such stencils to each type of anisotropy present in the BRDF independently. Our model allows for the fast direct fitting of parameters without the need of any demanding optimization. Furthermore, it achieves an encouraging, expressive visual quality in comparison to rival solutions that rely on a similar number of parameters. We believe that our method represents a promising approach to the analysis and modeling of complex anisotropic BRDF behavior.

Keywords anisotropic · highlight · stencils · BRDF · model

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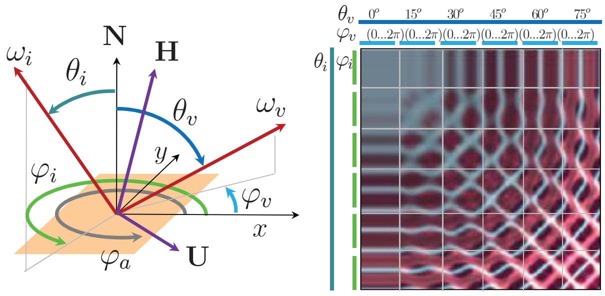


Fig. 1 BRDF angular parameterization (left) and BRDF unwrapping into 2D image (right) used within this paper.

1 Introduction

Each material surface reflects a certain portion of incident light. Generally, the light scatters in the material structure and leaves it either as reflectance or as transmittance. If we restrict the surface in being opaque, light is reflected in directions constrained by a hemisphere whose pole intersects with the surface normal.

The distribution of the differential reflected radiance dL for incident irradiance dE is a four-dimensional Bidirectional Reflectance Distribution Function (BRDF) [12]

$$B(\theta_i, \varphi_i, \theta_v, \varphi_v) = \frac{dL(\theta_v, \varphi_v)}{dE(\theta_i, \varphi_i)} = \frac{dL(\theta_v, \varphi_v)}{L(\theta_i, \varphi_i) \cos\theta_i d\omega_i}, \quad (1)$$

where parameterization of illumination ω_i and viewing ω_v directions is illustrated in Fig. 1. This equation defines anisotropic BRDF, i.e., having variable values when mutually fixed illumination and viewing directions are rotated around the material normal. When these values are constant, the BRDF is considered isotropic and defined as only a three-dimensional function. Fig. 1-right illustrates a 4D BRDF unfolded into 2D image, i.e., into individual BRDF subsets for increas-

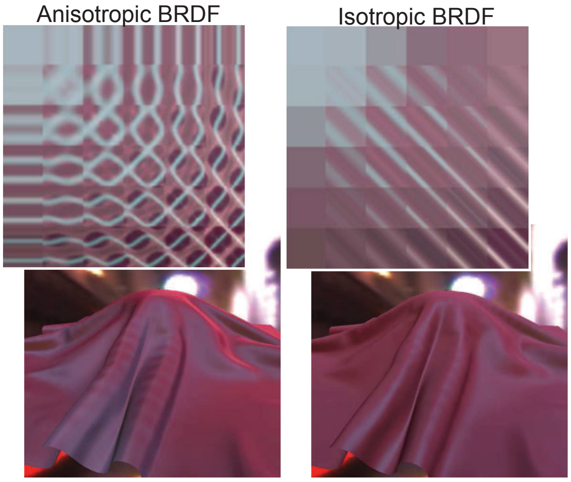


Fig. 2 Importance of anisotropic appearance. A comparison between BRDFs and rendering of the anisotropic material *fabric106* (left) and the same material with its anisotropic behavior removed (right) on 3D geometry in global illumination.

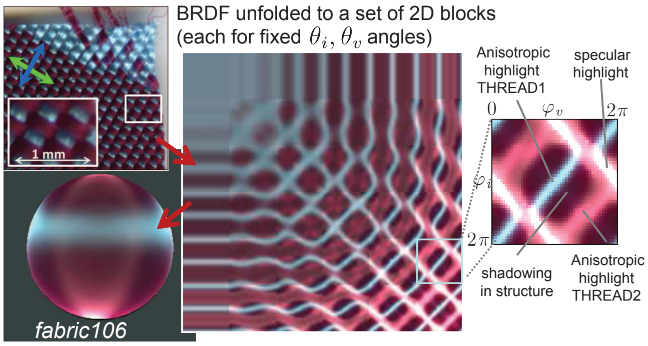


Fig. 3 An example of anisotropic BRDF behavior of *fabric106*. Major visual features are shown in BRDF subset obtained for fixed elevation angles. It illustrates azimuthally-dependent BRDF behavior.

ing elevation illumination / view angles from 0^o to 75^o are shown side by side vertically / horizontally.

Fig. 2 shows effect of anisotropic highlights when compared to isotropic highlights only; this example clearly demonstrates importance of modeling a full 4D anisotropic BRDF for highly anisotropic samples.

A BRDF has three important properties namely reciprocity, energy-conservation, and non-negativity. All of them are based on physical principles of light interaction with the surface. Helmholtz reciprocity states that illumination and viewing directions can be swapped without any effect on the BRDF value. Energy conservation specifies that the total reflected energy cannot be greater than the sum of incoming energy. Non-negativity ensures positive distribution values.

An example of a BRDF having two distinct anisotropic modes, i.e., unique highlights, is shown in Fig. 3. A detailed analysis of the selected subset reveals two different anisotropic highlights corresponding to two threads occurring in the material. While the brighter anisotropic highlight (AHL1) corresponds to a visually dominant thread (usually the upper one), the less bright one (AHL2) belongs to a less dominant thread (usually the bottom one). Note also increased intensity at specular direction, i.e., when $|\varphi_i - \varphi_v| \approx \pi$.

A main goal of BRDF modeling has been to develop a compact representation of sparsely measured BRDF data. In contrast to BRDF compression, the goal of BRDF models is, not only to compress measured samples, but also to predict values between the measured samples in order to approximate unknown data. The primary aim in optimal BRDF model development is in finding a compact parametric representation that can faithfully describe the dominant behavior of the BRDF. Moreover, such a model should be physically plausible, reciprocal and energy-conserving, as well as easy to implement in graphics hardware for higher efficiency.

A typical solution relies on an iterative algorithm that monotonically converge to a correct solution. However, this process is often very time demanding, and its results depend on a proper setting of initial values. The main contributions of this paper are (1) the analysis of typical behavior of anisotropic highlights using highlights stencils and (2) a new approach to BRDF modeling based on the direct fitting of these stencils to individual anisotropic highlights (related to a specific structure part of material surface). Our approach is not based on the global fitting of reflectance lobes parameters, but instead approximates individual modes of anisotropy independently, i.e., always using just a subset of BRDF. Therefore, it allows for the very fast identification of parameters without them needing to be initialized. Moreover, many parameters are intuitive, which thus allows for an efficient editing of fitted BRDFs.

The contents of the paper are as follow: In Section 2, we set our work in the context of BRDF modeling research. Section 3 describes the proposed BRDF model and Section 4 shows achieved results on a set of BRDFs. Section 5 discusses the method's pros and cons, while Section 6 concludes the paper.

2 Prior Work

In the past, BRDF measurements were approximated by several methods that exploited properties of typical BRDFs such as reciprocity, smoothness, location and shape of specular reflection, etc. We can roughly divide BRDF models into two categories. While empirical models compromise accuracy and physical plausibility in order to achieve a low number of parameters and thus faster evaluation, the physically derived models offer higher descriptive qualities, albeit at the cost of computational demands.

Most of empirical models represent BRDF by means of a specific type of reflectance function expressing a mutual relationship between the directions to illumination φ_i, viewer φ_v, and the normal direction $\mathbf{N}$ (see Fig. 1). The empirically derived models are usually based on a very simple formula with several adjustable parameters designed to fit a certain class of reflectance functions. The most common is a model by Phong [14] and its more practical modification by Blinn [3]. A generalization of the Phong model based on cosine lobes was introduced by Lafortune et al. [9]. A simplified physically plausible anisotropic reflectance model based on Gaussian distribution of micro-facets was presented by Ward [21]. The model has the necessary bidirectional characteristics and all four of its parameters have physical meaning; therefore, it can be fitted independently to measured BRDF data to produce a physically valid reflectance function that fulfills reciprocity and energy conservation. A model by Schlick [19] stands halfway between empirical and theoretical models. Ashikhmin et al. [1] extended the Phong-based specular lobe, made this model anisotropic, and incorporated a Fresnel behavior while attempting to preserve the computational simplicity of the initial model in addition to physical plausibility.

In contrast, physically motivated models are designed to represent some known physical phenomena; therefore, individual parameters or fitted functions are related to properties of real materials. Torrance and Sparrow's [20] pioneer model represents a surface by the distribution of vertical V grooves–perfectly, specular micro-facets. This model was later enhanced and introduced to the computer graphics community by Cook and Torrance [4], and further extended for more accurate and stable fitting by a shifted gamma micro-facet distribution in [2]. The micro-facet models are typically based on a combination of Fresnel, facet-distribution, and shadowing/masking functions. It was shown [7] that the distribution and masking functions can be effectively used for modeling of anisotropic behavior. Such an anisotropic extension of Cook-Torrance model was recently introduced by Kurt et al. [8]. This model is based on a normalized micro-facet distribution function, is physically plausible, and is designed to be more advantageous for data fitting and real-time rendering. The most complete and complex BRDF model was proposed by He et al. [6]. Although it accounts for many physical phenomenon involved in light reflection on rough surfaces such as polarization, diffraction, conductivity, and subsurface scattering, it cannot correctly represent anisotropic BRDFs. Oren–Nayar [11] extended Torrance and Sparrow [20] to modeling of surface roughness using Lambertian facets. An efficient isotropic BRDF model based on rational function has been proposed by Pacanowski et al. [13] recently. The model uses reduced Rusinkiewicz parameterization [17], relies on a compact set of parameters, and its extension allows limited modeling of anisotropic effects.

Another physically motivated appearance model by Sadeghi et al. [18] extending original work of Poulin and Fournier [15], is based on the physics of light scattering in cylindrical threads of fibers and achieves anisotropy by the application of a user defined weaving pattern of two threads together with parameters of their reflectance model. A drawback of this model is a complicated application of parameter optimization techniques; therefore, derivation of appropriate parameters of individual threads is left completely to users and their experience.

Our approach is closely related to anisotropic empirical models, yet instead of the iterative fitting of a global reflectance, it directly represents anisotropic phenomena caused by individual directionally dependent structure elements. This results in a fast and robust fitting to BRDF data using a compact parameter set comparable in size with competing models and with minimal user input requirements.

3 The Proposed BRDF Model

The proposed model is based on independent modeling of anisotropic modes in the material, e.g., individual threads in case of fabrics or fibers in case of wood. Although the proposed model is not limited to modeling of two anisotropy modes only we restrict ourselves so in this paper.

3.1 Model Description

An overall scheme of the proposed model is outlined in Fig. 4. We describe individual parts of the model in the following sections with letters related to those in the scheme. Note that the products of the model dependent on incoming and outgoing directions are denoted by capital letters, while scalar parameter values are lowercase or in Greek letters.

A. *Anisotropic highlights prediction*

Our method starts with representation of main anisotropy axes using highlights stencils. First, we automatically detect the azimuthal location of anisotropic highlights (i.e., φ_a of each thread) by analyzing cross-sections

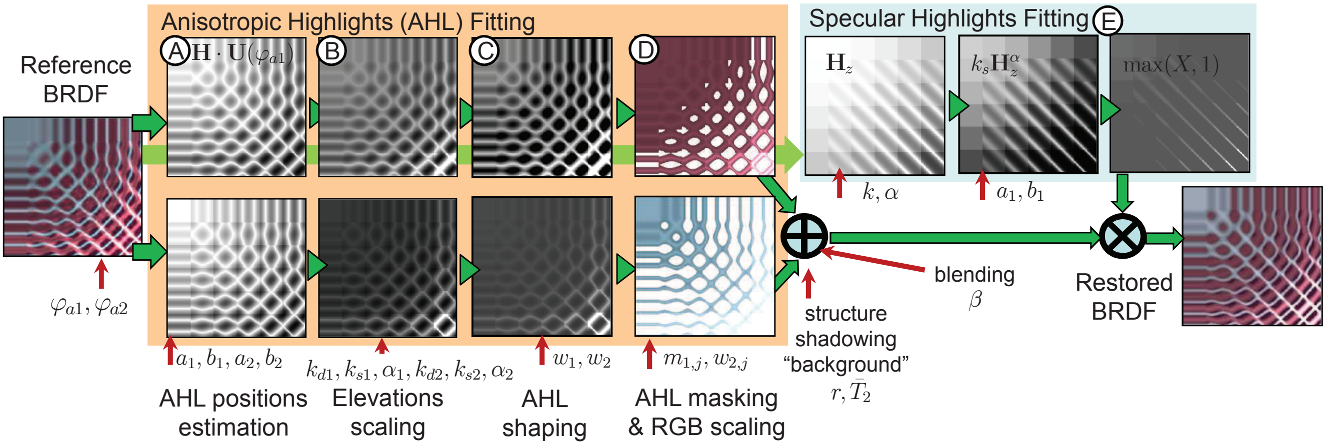


Fig. 4 Overall scheme of the proposed BRDF model.

of BRDF for fixed elevation illumination direction ($\theta_i = 75^o, \varphi_i = 0^o, \theta_v \approx 60^o$). As we expect two modes of anisotropy, we are looking just for two peaks in azimuthal angles ($0 - -2\pi$), and their location determines direction of anisotropy φ_a as shown in Fig. 5.

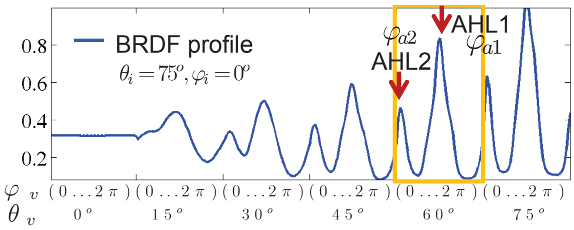
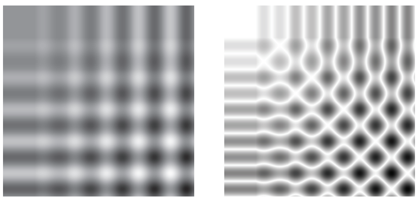


Fig. 5 Detection of anisotropic directions φ_{a1} and φ_{a2} from 1D BRDF profile. Only data in yellow frame are needed.

Then the locations of anisotropic highlights can be predicted on directional surface elements, laying in planes orthogonal to the bisector (often denoted as half-way direction $\mathbf{H} = \frac{\omega_i+\omega_v}{||\omega_i+\omega_v||}$) of directions of incidence and reflectance as proposed in [10]. Therefore, the anisotropic highlights can be predicted [16] at directions of the bisector which are orthogonal to the detected anisotropy axis of the material $\mathbf{U} = [\sin\varphi_a, \cos\varphi_a, 0]$ (see Fig. 1). Thus, the highlights stencils can be expressed as

$$F(\varphi_a) = 1 - |\mathbf{H} \cdot \mathbf{U}(\varphi_a)| \qquad (2)$$
$$\mathbf{U}(\varphi_a) = [\sin\varphi_a, \cos\varphi_a, 0]$$

Fig. 6-b shows the application of the highlights stencil function F for prediction of anisotropic highlights locations across many incidence and reflectance directions for a given φ_a.



(a) $\mathbf{H} \cdot \mathbf{U}(\varphi_a) + 1$ (b) $1 - |\mathbf{H} \cdot \mathbf{U}(\varphi_a)|$

Fig. 6 Highlights stencil: predicting of anisotropic highlight location (b) using half-way direction $\mathbf{H}$ and the detected direction of anisotropy $\mathbf{U} = [\sin\varphi_a, \cos\varphi_a, 0]$, where $\varphi_a = 0^o$.

B. *Elevation dependent scaling*

The obtained stencil functions F for each thread (φ_{a1},φ_{a2}) have to be scaled to reflect illumination/view dependent intensity present in the to-be-fitted BRDF. Therefore, the anisotropic highlight for respective threads is sampled once at each view/illumination elevation angle across entire BRDF to acquire its elevation dependent intensity (see blue and green dots in Fig. 7). This

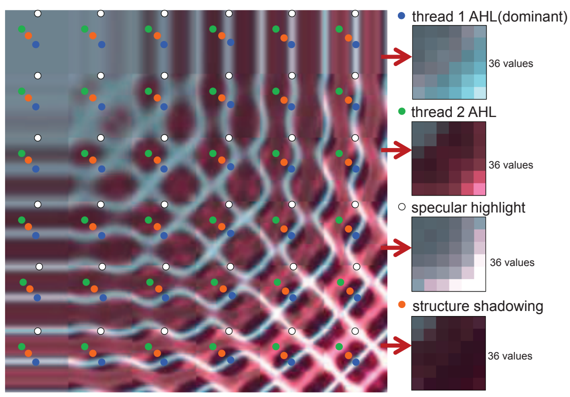


Fig. 7 Data collection for model fitting:sampling of anisotropic highlights (blue and green dots), specular highlights (white dots), and shadowed areas (orange dots) values.

2D function representing elevation-dependent intensity is approximated by parameters a, b and applied to scale functions $F(\varphi_{a1}), F(\varphi_{a2})$ as follows

$$E_1 = F(\varphi_{a1}) \left(a_1 + \frac{b_1}{\cos\theta_i \cos\theta_v} \right) \quad (3)$$

$$E_2 = F(\varphi_{a2}) \left(a_2 + \frac{b_2}{\cos\theta_i \cos\theta_v} \right) \quad (4)$$

C. *Anisotropic highlights shape fitting*

The shape of the predicted anisotropic highlights in functions E_1 and E_2 is further adapted to the BRDF data, due to its simplicity, by a variant of the Blinn model [3]. To this end, we fit model parameters (diffuse and specular parameters k_d, k_s, and specular exponent α) for each thread (further discriminated by numbers 1 and 2) using the luminance of the to-be-fitted BRDF and the function E. To account for anisotropic highlights areas only in our fitting, we generate support masks M around each thread's highlight as shown in Fig. 4-D. The mask for each thread includes only a limited surrounding along its anisotropic highlights and thus is used for identification of thread's support directions for a proper k_d, k_s, α fitting and for further blending. As the function $\mathbf{H} \cdot \mathbf{U}$ represents modulated sinusoidal function (see Fig. 6-a), the masks can be obtained using thresholding of function $|\mathbf{H} \cdot \mathbf{U}(\varphi_a)| < w = 1 - \sin(\sigma/2)$, where σ is width of specular highlight (in degrees) estimated from the fitted BRDF.

Finally, color information is introduced by the scaling of a fitted luminance value using 3 RGB color parameters m_i.

$$T_{1i} = m_{1i} \cdot \max(0, k_{d1} + k_{s1} E_1^{\alpha_1}) \quad (5)$$

$$T_{2i} = m_{2i} \cdot \max(0, k_{d2} + k_{s2} E_2^{\alpha_2}) \ , \quad (6)$$

where i stands for color channel index and minimum guarantees BRDF non-negativity. The color parameters m_i are obtained by the fitting of the luminance function $k_d + k_s E^\alpha$ to values of individual color channels in the BRDF.

D. *Threads masking and blending*

At this point each thread is represented by its support contours and intensity in each color channel. The information from both threads have to be blended and the missing parts of the BRDF filled. Dark reflectance values not belonging to highlight areas corresponds to the "background" color related to shadowing mainly in the structure of the less dominant thread. Therefore, we pick the darkest measured value for each illumination/view elevation in BRDF and compute the linear scaling factor r between the color intensities of the "background" and the average color of the less dominant thread ($r < 1$).

If the support masks of the threads intersect, i.e., $M_{\varphi_{a1}}(w_1) \cap M_{\varphi_{a2}}(w_2)$, we combine threads information as follows

$$B_i = \beta M_{\varphi_{a1}}(w_1) T_{1i} + (1 - \beta) M_{\varphi_{a2}}(w_2) T_{2i} \ , \quad (7)$$

where parameter β controls a final contributions of both threads at the intersections. If $\beta = 0$, the upper thread is considered fully opaque, while its increasing value introduces its translucency. For incoming and outgoing directions, where only one of the threads contributes, we use a reduced form

$$B_i = M_{\varphi_{a1}}(w_1) T_{1i} + M_{\varphi_{a2}}(w_2) T_{2i} \ . \quad (8)$$

And finally, for directions where there is no thread contribution, we render only background color, i.e., a mean color of the less dominant thread scaled by parameter $r < 1$, i.e.,

$$B_i = r \cdot \bar{T}_{2i} \ . \quad (9)$$

E. *Specular highlights fitting*

As the majority of the materials also have a significant specular contribution on the top of the anisotropic highlights represented by modified stencil functions, we approximate this behavior by the modification of the z-coordinate of half-vector direction H_z. As the upper thread represents the main contribution to the specular highlight, we approximate specular highlights elevation-dependency by means of the upper thread's intensity, i.e., using parameters a_1, b_1 for introduction of specularity elevation dependency, while the shape of specular highlights is controlled using the Blinn model again (parameters k, α). Specular highlights are sampled similarly as anisotropic highlights, i.e., one sample for each combination of illumination/viewing elevation angles. Additionally, we also sample the darkest value for each elevations combinations (see white and orange dots in Fig. 7). This data and the elevation intensity compensated function H_z are used to fit the model's parameters.

Finally, specular contribution is added to our model as follows

$$B_{S,i} = B_i \cdot \max \left[1, k \cdot H_z^\alpha \cdot \left(a_1 + \frac{b_1}{\cos\theta_i \cos\theta_v} \right) \right] \ . \quad (10)$$

Maximum function prevents the specular factor to decrease the final reflectance intensity.

3.2 Implementation Details

Parameters – Our model has 10 parameters per thread: anisotropic axis direction φ_a, width of thread support w, elevation-dependent intensity fitting a, b, color scaling $3 \times m$, and parameters controlling shape of the

Table 1 Number of parameters of individual tested models.

parameters	Kurt [8]	Sadeghi [18]	*proposed*
global	3	0	4
per thread/lobe	7	min. 10	10
total (2 th./lob.)	17	min. 20	24

thread (k_d, k_s, α). Additionally, it has four global parameters: blending factor β, background scaling factor r, and specular highlight modeling parameters k, α. A comparison of parameters count required by several anisotropic BRDF modeling approaches is given in Tab. 1.

Data fitting – The direct parameters estimation we applied is data driven and avoids iterative optimization dependent on initial values; therefore, it can be captured in a local minimum. As no iterative optimization is applied in our method, the proposed fitting is very fast. It takes only 3 s using a single core of Intel Core i7. We used standard MATLAB functions robustfit for linear parameter estimation and nlinfit for nonlinear parameters estimation.

A minimal variant of our method requires only $4 \times 36 = 144$ color BRDF samples (see Fig. 7) to estimate its parameters, however, an optimal shape reproduction of specular and anisotropic highlights requires more samples along these features. Note that although location of samples shown in Fig. 7 is angularly uniform, we assume that our method would be able to learn its parameters also from scattered BRDF samples of reasonable density. This can be done either by taking the closest value, or by interpolating possibly missing values at some elevations.

The adjustment of three parameters is up to the user; although, their default values work fairly well. Blending factor β controls model behavior at directions of threads intersections. The typical value is 0, although it can be increased up to 1 in relation to upper thread's translucency. Support of the threads is controlled by their width parameter $w = 1 - \sin(\sigma/2)$, where 0 represents extremely wide, while 1 extremely narrow threads. Although we estimate the angle σ from the BRDF, the parameter w often requires a slight user adjustment to achieve the best fitting performance. Typical values is w_1=0.7 for upper thread and w_2=0.6 for the bottom one, as the bottom thread has often wider highlight support.

Note that the model parameters for all materials in the paper and supplemental material were obtained automatically, without any manual intervention. Only parameters β, w_1, w_2 were slightly adjusted for each material. Please see the list of all model parameter values for each material in the supplemental material. The model's performance can be further improved by the user slightly tuning it. Our implementation in MATLAB is publicly available[1].

4 Experiments

4.1 Test Datasets

As the number of publicly available anisotropic BRDFs is limited we resorted to the UTIA BRDF Database[2] [5] as the only source of reasonable number of different types of anisotropic BRDF measurements. This database contains 150 BRDF out of which over 50 exhibit some aspect of anisotropy. The BRDFs are stored in 32bit float HDR format and their angular resolution is 15^o in elevations and 7.5^o in azimuthal angles. We tested our model on fourteen BRDFs exhibiting various types of anisotropy (13 types of fabric, 1 wood).

4.2 Results

We fitted our model to 14 tested BRDFs. Fig. 8 shows the results for four of them. The figure compares a reference BRDF measurement with its reconstruction using two physically-based anisotropic BRDF models by Kurt et al. [8] and Sadeghi et al. [18]. In the Kurt model we fitted two lobes each using dedicated parameters k_s, k_d for each RGB channel, while lobe parameters f_0, m_x, m_y, α were fitted to a luminance values only. The Sadeghi model was fitted by a parametrization of two threads as suggested in the paper. We included difference images between reference and reconstructed BRDFs as well as mean difference computational values using ΔE / RMSE / PSNR [dB] / SSIM / VDP2 metrics. The results demonstrate an encouraging performance in our model especially when compared with the competing methods.

This is supported by a comparison of a BRDFs rendering on a frontally illuminated sphere as shown in Fig. 9, where our model shows a promising performance in capturing of most anisotropic features.

Finally, Fig. 11 presents side-by-side renderings of reference BRDF and its reconstruction using the proposed model for all fourteen tested BRDFs in a global illumination environment. The remaining results including parameters listing are shown in the supplementary material.

[1] http://anonymous
[2] http://btf.utia.cas.cz

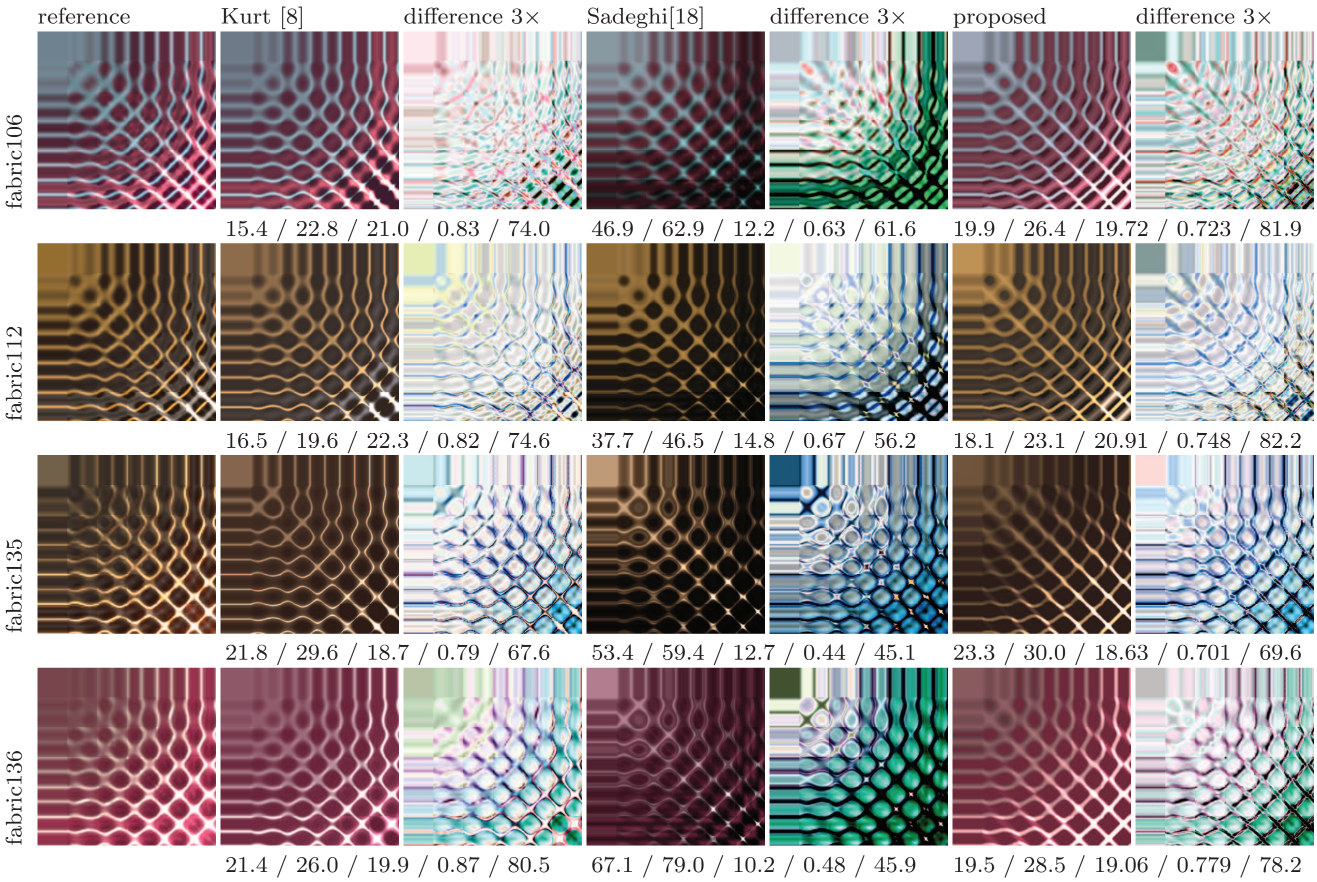


Fig. 8 A comparison of reference BRDF and its reconstructions using three compared models for four BRDFs of anisotropic fabric materials. Included are difference images (scaled 3×) and results of ΔE / RMSE / PSNR [dB] / SSIM / VDP2 metrics.

5 Discussion

5.1 Advantages

A notable advantage of our model is a fast and reasonably robust fitting procedure without the need for demanding optimization.

As our approach is based on the direct prediction of individual anisotropic modes, it is well suited to the interactive editing of BRDFs, perhaps utilized in future intuitive GUI. Fig. 12 illustrates several straightforward editing examples – change of thread's anisotropy axes, width of anisotropic highlight, and thread's color. Note that such edits are possible due to the fact that model parameters are not fitted globally, but instead estimated independently for individual anisotropic modes.

Our anisotropic highlights prediction algorithm can be also applied to BRDF retro-reflection modeling as demonstrated in Fig. 10. The retro-reflective highlights can be predicted merely by modifying azimuthal illumination angles $\varphi_i = \varphi_i + \pi$ prior to half-way direction $\mathbf{H}$ computation for highlights stencils predictions.

5.2 Limitations

The proposed modeling method naturally preserves reciprocity in BRDF, while the BRDF non-negativity is enforced. As for majority of empirical models energy conservation in BRDFs reconstructed by our method cannot be guaranteed due to possible inaccuracy in specular component fitting.

A minor drawback in our approach is the requirement of a slight user intervention to adjust a global model's parameters (transparency β, highlights width w) so as to achieve optimal results. Also our current version of automatic parameters detection tends to shift color hue of the BRDF; however, the simple tuning of parameters m_r, m_g, m_b can effectively remedy this. Usage of a current version of our model in Monte-Carlo importance sampling schemes is limited due to a non-linear model equation.

Although our current implementation assumes only two modes of anisotropy (e.g., two threads), we consider its extension to modeling of more complex anisotropic behavior possible and leave it for our future endeavor.

Fig. 11 Comparison between reference BRDF (odd rows) and proposed BRDF model (even rows) renderings in *grace* illumination environment for 14 tested BRDFs.

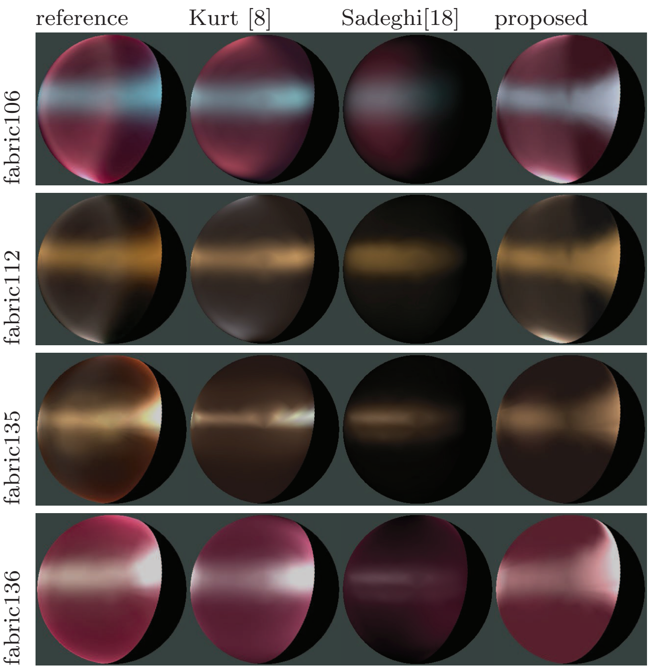


Fig. 9 A rendering comparison of reference BRDF and its reconstructions, on sphere illuminated by point light from left, using three compared BRDF models.

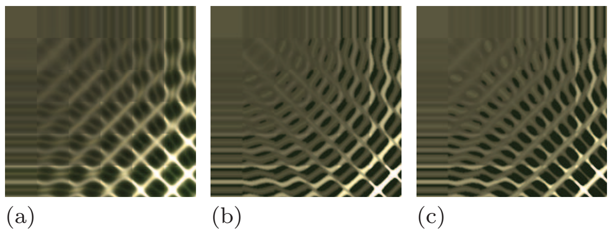


Fig. 10 Extension to retro-reflection highlights modeling: (a) reference BRDF, reconstructed BRDF (b) with anisotropic highlights, (c) with retro-reflective highlights.

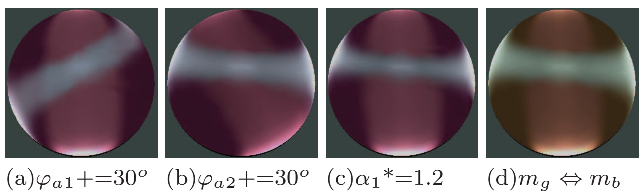


Fig. 12 Examples of BRDF anisotropy editing: (a) AHL1 shift 30^o, (b) AHL2 shift 30^o, (c) AHL1 width narrowed, (d) threads color parameters swapped.

6 Conclusions

We present a novel empirical anisotropic BRDF model that fits individual modes of anisotropy independently using adaptive stencils of anisotropic highlights. Therefore, our approach does not need any time demanding and potentially unstable numerical optimization. We tested performance on fourteen BRDFs exhibiting a wide range of anisotropic behavior. Our model allows for a very fast fitting, achieving robust results while simultaneously minimize the need for user interaction. Furthermore, due to the independent fitting of individual modes of anisotropy, the proposed model allows for an intuitive editing of anisotropic appearance.

We consider our model suitable for GPU rendering, however, the GPU implementation is still a subject of our future work. Similarly, we plan to extend our model to cope with even more complex anisotropic behavior, e.i., more than two anisotropy modes.

Acknowledgments

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